Measuring New Hampshire's Tamworth Big Pine
By Robert T. Leverett

(Prepared for the Committee and David Govatski as part of the
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Introduction

On September 23, 2023, as part of the New Hampshire Old Growth Conference agenda, the team of Erik Danielsen, Jared Lockwood, Dale Luthringer, and Bob Leverett volume-modeled the trunk of a huge pine growing in Big Pines Natural Area of New Hampshire's Hemenway State Forest. The purpose of the modeling was to estimate the amount of carbon held within the pine and its CO_2 equivalent. In addition to providing a description of the event, this report is also motivated by the on-going debate about the importance and role of trees, and especially big ones, in mitigating climate change.

All members of the measuring team belong to the **Native Tree Society (NTS)** and **American Forests Champion Tree Certification Cadre**. In addition, Bob Leverett and Jared Lockwood are members of the National Champion Tree Measuring Guidelines team.

What follows is a report on the team's efforts on Sept 23rd to measure the trunk volume of the Tamworth Big Pine. The results, included as an appendix, are followed by comparisons to twelve statistical volume-biomass models for white pine available through the U.S. and Canadian Forest Services and other sources. The report concludes with an evaluation of the Big Pine's contribution to storing carbon as compared to other pines.

Tamworth Big Pine

The huge tree that we call Tamworth Big Pine measures 15.2 feet in circumference at 4.5 feet above mean base level. Erik Danielsen measured the pine's height at 158.8 feet. These dimensions place the tree among the largest that we have measured in New England for a forest-grown pine. We don't know the age of the pine but estimate that it is at least 175 years old and probably older. A natural question to ask is how many board feet there are in the tree, and as an extension, what is the pine's total above ground volume? Another question is how much carbon does the pine hold in its trunk and limbs? This last question has relevance to climate change. But how could we estimate carbon without cutting the tree down, sectioning it, kiln-drying the sections, and weighing each? Fortunately, we don't have to destructively sample the pine. We can measure it as it stands and compute its volume, dry biomass, and carbon component. This is exactly what we did on September 23rd.

The Method of Measurement and Results

The measuring process for a standing tree involves the following steps:

1. Divide the trunk into sections and measure the volume of each separately.
    a. Each section is modeled as a regular geometric solid
    b. Adjustments can be made for sections that are not circular in cross-section.
2. Add up the section volumes
3. Multiply the total volume by the dry weight density of the trunk expressed in lbs/ft^3. Densities from Miles & Smith are provided separately for bark and wood. The volume is in terms of the components when green. For example, a cubic foot of green volume holds so much dry biomass. The rest is water, which is not counted. The process of computing weight of the dry wood and bark in a green cubic foot will be given later.
4. The result of step 3 is dry biomass, which is 52.1% carbon by weight. Simply multiplying the biomass by 52.1% gives the carbon weight.
5. The carbon deposited in the trunk, branches, foliage, and roots comes completely from atmospheric CO_2. The ratio of the atomic weight of a CO_2 molecule to its carbon atom is 3.666. Therefore, if we multiply the weight of the carbon that we computed in the Tamworth Big Pine by 3.666, we'll get the amount of CO_2 taken from the atmosphere through photosynthesis to yield the computed weight of carbon.
6. We ignore the foliage because it stays on the tree only a few years and when on the ground, it decays rapidly, returning its CO_2 to the atmosphere.
7. The volume of the limbs is computed through a formula that is based on diameter at breast height. The resulting volume is then converted to dry biomass and then to carbon and CO_2 following the computational process used for the trunk.
8. Roots are added in as a percentage of above ground biomass.

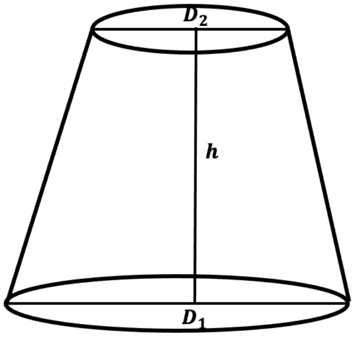
The trunk volume measured for the Big Pine was 792 ft^3. The trunk volume, excluding the stump and the top above a 4-inch outside bark diameter, is approx. 750 ft^3. Theoretical board feet is 9,000, but that includes bark. Stem wood calculates to 7,560 board feet. The usable part of this is probably between 50% and 60%. At 60%, we have 4,536 board feet.

Addition of limb volume through an allometric equation gives 913.0 ft^3. This is green volume. The above-ground dry biomass contained in this green volume approximately equals 22.33 x 913.0 = 20,387.3 lbs. Of this biomass, 52.1% is elemental carbon, giving 10,621.8 lbs or 5.3 regular tons. The final step is to convert this carbon to CO_2 equivalent, which is the amount of CO_2 taken out of the atmosphere through photosynthesis. We multiply the 10,621.8 lbs by 3.666 to get 38,939.5 lbs. We can add approximately 15% for the CO2 that went into developing the underground root system. The final figure is 44,780.4 lbs. This last amount will continue to rise so long as the pine is reasonably healthy, but at what rate, we cannot say.

The question that may be fairly raised is how good are the above estimates? We will be developing a set of appendices. In them, we intend to describe the methods used in the above analysis. We will also compare our direct trunk volume measurement to what comes from applying different statistical models. The final appendix will predict future growth and discuss the attendant climate implications.

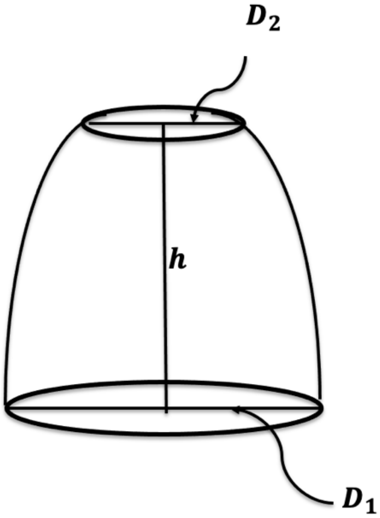
Appendix I: Regular Geometric Frustums Used to Model Trunk Sections

The method we used to determine the volume of the Tamworth Pine required that we divide the trunk into adjoining sections. Each section was then modeled as the frustum of a regular geometric solid. The cross-sectional area of each segment was treated as circular, although we have ways of handling non-circular cross-sections. If the taper from the base to the top was constant, then the section could be modeled as a frustum of a right circular cone. The following diagram depicts such a frustum.

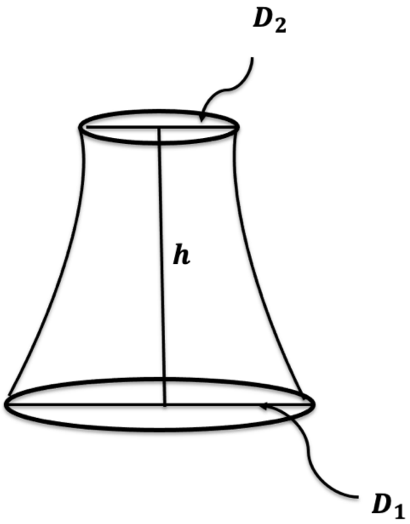


Here D_1 = diameter of base, D_2 = diameter of top, and h = height of frustum.

If the sides were convex, we could model the section as a frustum of a right circular paraboloid. See the following figure.



If the sides were concave, we could model the section as a frustum of a right circular neiloid. See the following figure.



The volume equations for the three above forms follow.

Conical frustum

$$V = \frac{1}{12}\pi h(D_1^2 + D_1 D_2 + D_2^2)$$

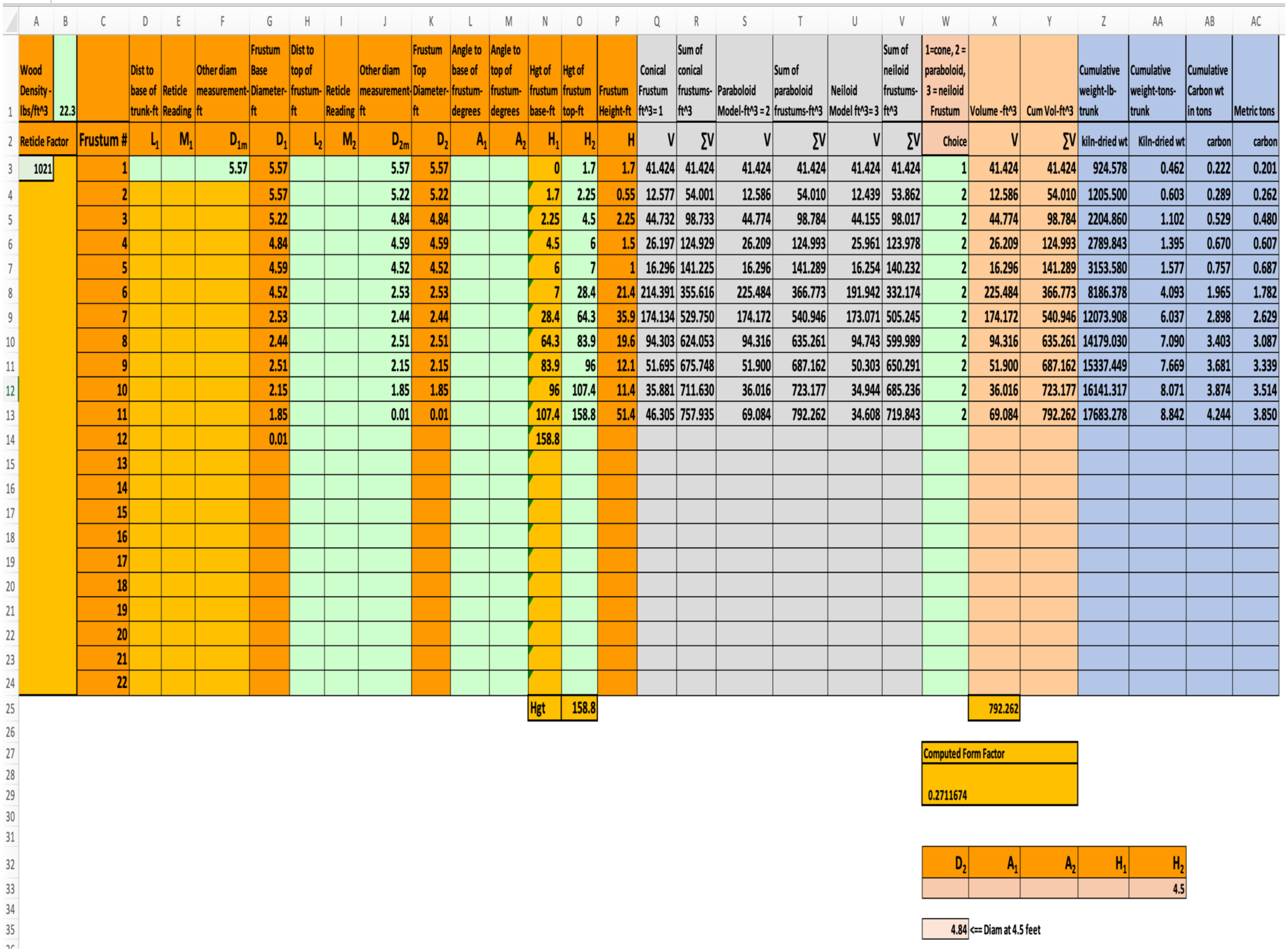
Paraboloid frustum

$$V = \frac{1}{8}\pi h(D_1^2 + D_2^2)$$

Neiloid frustum

$$V = \frac{1}{16}\pi h\left(D_1^2 + D_1^{\frac{2}{3}} D_2^{\frac{4}{3}} + D_1^{\frac{4}{3}} D_2^{\frac{2}{3}} + D_2^2\right)$$

The above choices were built into an Excel worksheet. An image of the worksheet follows that contains all the measurements and accompanying calculations.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Wood Density -lbs/ft^3	22.3		Dist to base of trunk-ft	Reticle Reading	Other diam measurement-ft	Frustum Base Diameter-ft	Dist to top of frustum-ft	Reticle Reading	Other diam measurement-ft	Frustum Top Diameter-ft	Angle to base of frustum-degrees	Angle to top of frustum-degrees	Hgt of frustum base-ft	Hgt of frustum top-ft	Frustum Height-ft
2	Reticle Factor		Frustum #	L_1	M_1	D_{1m}	D_1	L_2	M_2	D_{2m}	D_2	A_1	A_2	H_1	H_2	H
3	1021		1			5.57	5.57			5.57	5.57			0	1.7	1.7
4			2				5.57			5.22	5.22			1.7	2.25	0.55
5			3				5.22			4.84	4.84			2.25	4.5	2.25
6			4				4.84			4.59	4.59			4.5	6	1.5
7			5				4.59			4.52	4.52			6	7	1
8			6				4.52			2.53	2.53			7	28.4	21.4
9			7				2.53			2.44	2.44			28.4	64.3	35.9
10			8				2.44			2.51	2.51			64.3	83.9	19.6
11			9				2.51			2.15	2.15			83.9	96	12.1
12			10				2.15			1.85	1.85			96	107.4	11.4
13			11				1.85			0.01	0.01			107.4	158.8	51.4
14			12				0.01							158.8		
15			13													
16			14													
17			15													
18			16													
19			17													
20			18													
21			19													
22			20													
23			21													
24			22													
25														Hgt	158.8	

	C	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC
1		Conical Frustum ft^3=1	Sum of conical frustums-ft^3	Paraboloid Model-ft^3 = 2	Sum of paraboloid frustums-ft^3	Neiloid Model ft^3= 3	Sum of neiloid frustums-ft^3	1=cone, 2 = paraboloid, 3 = neiloid Frustum	Volume -ft^3	Cum Vol-ft^3	Cumulative weight-lb-trunk	Cumulative weight-tons-trunk	Cumulative Carbon wt in tons	Metric tons
2	Frustum #	V	∑V	V	∑V	V	∑V	Choice	V	∑V	kiln-dried wt	Kiln-dried wt	carbon	carbon
3	1	41.424	41.424	41.424	41.424	41.424	41.424	1	41.424	41.424	924.578	0.462	0.222	0.201
4	2	12.577	54.001	12.586	54.010	12.439	53.862	2	12.586	54.010	1205.500	0.603	0.289	0.262
5	3	44.732	98.733	44.774	98.784	44.155	98.017	2	44.774	98.784	2204.860	1.102	0.529	0.480
6	4	26.197	124.929	26.209	124.993	25.961	123.978	2	26.209	124.993	2789.843	1.395	0.670	0.607
7	5	16.296	141.225	16.296	141.289	16.254	140.232	2	16.296	141.289	3153.580	1.577	0.757	0.687
8	6	214.391	355.616	225.484	366.773	191.942	332.174	2	225.484	366.773	8186.378	4.093	1.965	1.782
9	7	174.134	529.750	174.172	540.946	173.071	505.245	2	174.172	540.946	12073.908	6.037	2.898	2.629
10	8	94.303	624.053	94.316	635.261	94.743	599.989	2	94.316	635.261	14179.030	7.090	3.403	3.087
11	9	51.695	675.748	51.900	687.162	50.303	650.291	2	51.900	687.162	15337.449	7.669	3.681	3.339
12	10	35.881	711.630	36.016	723.177	34.944	685.236	2	36.016	723.177	16141.317	8.071	3.874	3.514
13	11	46.305	757.935	69.084	792.262	34.608	719.843	2	69.084	792.262	17683.278	8.842	4.244	3.850
14	12													
15	13													
16	14													
17	15													
18	16													
19	17													
20	18													
21	19													
22	20													
23	21													
24	22													
25									792.262					

Computed Form Factor

0.2711674

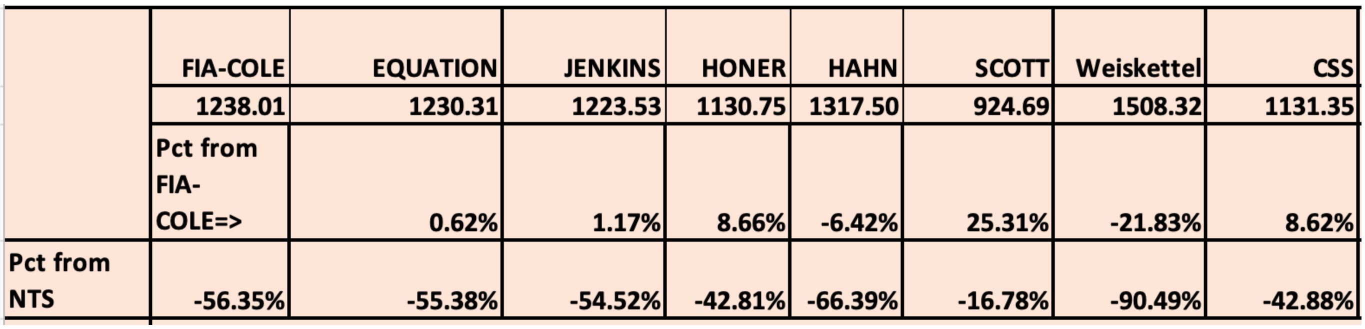
D_2	A_1	A_2	H_1	H_2
				4.5

4.84 <== Diam at 4.5 feet

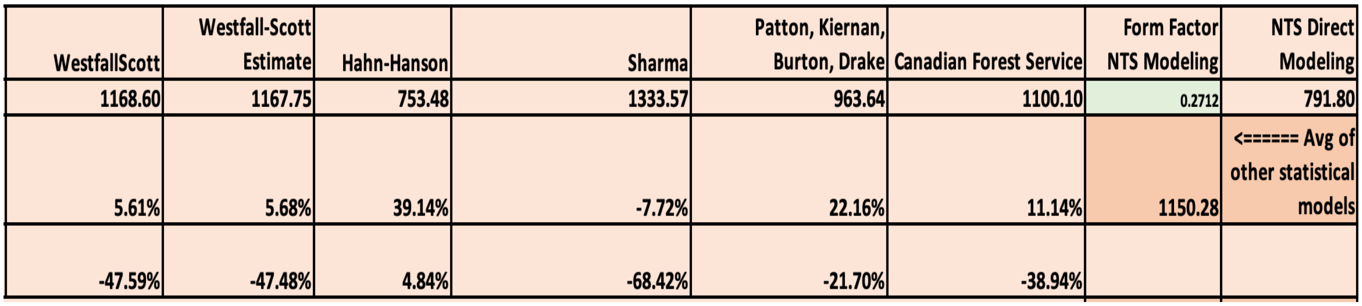
The calculated trunk volume is displayed in cell X25. A detailed explanation of this worksheet can be given on request, but the biggest surprise is the very low trunk form factor of 0.271. The expected factor for a white pine in the age class of the Tamworth Big Pine is in the range of 0.38 to 0.45 with 0.42 being the expected value. 0.42 would yield a trunk volume of 1,227 ft^3. Appendix II compares the direct volume modeling to returns of statistical volume-biomass equations from the U.S. and Canadian Forest Services.

Appendix II: Direct Measurement Compared to Statistical Models for Tamworth Big Pine

How do mainline statistical trunk volume models compare with the direct measurement volume modeling accomplished by NTS. The following two tables provide a comparison of direct measurement to 14 statistical models.



	FIA-COLE	EQUATION	JENKINS	HONER	HAHN	SCOTT	Weiskettel	CSS
	1238.01	1230.31	1223.53	1130.75	1317.50	924.69	1508.32	1131.35
	Pct from FIA-COLE=>	0.62%	1.17%	8.66%	-6.42%	25.31%	-21.83%	8.62%
Pct from NTS	-56.35%	-55.38%	-54.52%	-42.81%	-66.39%	-16.78%	-90.49%	-42.88%



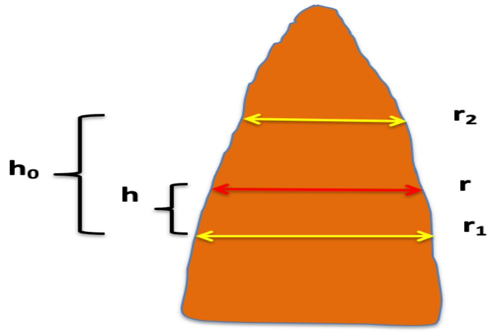
WestfallScott	Westfall-Scott Estimate	Hahn-Hanson	Sharma	Patton, Kiernan, Burton, Drake	Canadian Forest Service	Form Factor NTS Modeling	NTS Direct Modeling
1168.60	1167.75	753.48	1333.57	963.64	1100.10	0.2712	791.80
5.61%	5.68%	39.14%	-7.72%	22.16%	11.14%	1150.28	<====== Avg of other statistical models
-47.59%	-47.48%	4.84%	-68.42%	-21.70%	-38.94%		

As can be seen, the Tamworth Big Pine's volume as measured by NTS is much less than what is predicted by the statistical models. Erik Danielsen observed that the trunk between 7 and 28 feet exhibits unusual taper. A return to the Tamworth Big Pine will focus attention on trunk taper for the first 50 feet. We can compare the Big Pine's trunk taper to that predicted by the U.S. Forest Service Westfall-Scott Taper Model and equations for computing the radius at points within. frustum. See Appendix III: Trunk Taper Comparisons for Tamworth Big Pine.

Appendix III: Trunk Taper Comparisons for Tamworth Big Pine

Height above base-ft	Measured Diameter-ft	Westfall-Scott-ft (predicted)	Difference-ft (Measured – WS)
1.70	5.57	5.30	0.27
2.25	5.22	5.15	0.07
4.50	4.84	4.80	0.04
6.0	4.59	4.66	-0.07
7.0	4.52	4.59	-0.07
28.4	2.53	3.98	-1.45
64.3	2.44	3.36	-0.92
83.9	2.51	2.94	-0.43
96.0	2.15	2.57	-0.42
107.4	1.85	2.16	-0.31

Another visit to the Tamworth Pine is required to concentrate on trunk taper above 7 feet. In the interim, we can analyze the taper of the Tamworth Big Pine using a method that predicts radius (or diameter) at intermediate points within a frustum. It utilizes the following taper equations.



As an example, the diagram above shows a profile of a trunk that appears to be a paraboloid The area within the yellow arows is a frustum, which also appears to be a paraboloid.

$$r=\left[r_1^{1/p}+\left(r_2^{1/p}-r_1^{1/p}\right)\frac{h}{h_0}\right]^p$$

general taper equation

$$p=0.5$$

paraboloid factor

$$r=\left[r_1^{1/0.5}+\left(r_2^{1/0.5}-r_1^{1/0.5}\right)\frac{h}{h_0}\right]^{0.5}$$

$$r=\left[r_1^2+\left(r_2^2-r_1^2\right)\frac{h}{h_0}\right]^{0.5}$$

paraboloid equation

$$r=\left[r_1^{1/p}+\left(r_2^{1/p}-r_1^{1/p}\right)\frac{h}{h_0}\right]^p$$

general taper equation

$$p=1$$

cone factor

$$r=\left[r_1+\left(r_2-r_1\right)\frac{h}{h_0}\right]$$

cone taper equation

$$r=\left[r_1^{1/p}+\left(r_2^{1/p}-r_1^{1/p}\right)\frac{h}{h_0}\right]^p$$

general taper equation

$$p=1.25$$

General factor

$$r=\left[r_1^{0.8}+\left(r_2^{0.8}-r_1^{0.8}\right)\frac{h}{h_0}\right]^{1.25}$$

neiloid taper equation

Let’s construct a frustum from 7 feet up to 83.9 feet. Frustum height is 76.9 feet. Base radius is 2.26 feet. Top radius is 1.26 feet from direct measurement. Using the above taper equations, we can compute radius at heights of 28.4 and 64.3 feet above the ground. The following table gives the results.

Hgt above ground	Hgt above frustum base	Radius assuming paraboloid	Radius assuming cone	Radius assuming neiloid	Measured radius	Radius using Westfall-Scott	Closest predictor
28.4	14.4	2.109	2.073	2.065	1.27	1.99	WS
64.3	57.3	1.576	1.515	1.504	1.22	1.68	neiloid

These comparisons remind us that trees are individuals, and the larger, older ones can depart from statistical norms by a lot. A few reticle-based measurements at points up a trunk can alert us to a highly non-standard form if it isn’t apparent from visual inspection.